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# Contaminant Hydrogeology

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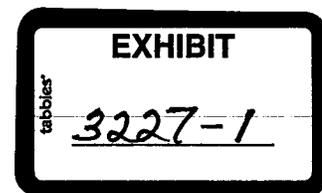
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Macmillan Publishing Company  
New York

Maxwell Macmillan Canada  
Toronto

Maxwell Macmillan International  
New York Oxford Singapore Sydney



Editor: Robert A. McConnin  
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Illustrations: Maryland CartoGraphics Inc.

This book was set in Garamond by Syntax International and was printed and bound by Book Press, Inc., a Quebecor America Book Group Company. The cover was stamped by Book Press, Inc., a Quebecor America Book Group Company.

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Printed in the United States of America

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Macmillan Publishing Company  
866 Third Avenue  
New York, New York 10022

Macmillan Publishing Company is part of the  
Maxwell Communication Group of Companies.

Maxwell Macmillan Canada, Inc.  
1200 Eglinton Avenue East, Suite 200  
Don Mills, Ontario M3C 3N1

**Library of Congress Cataloging-in-Publication Data**

Fetter, C. W. (Charles Willard)

Contaminant hydrogeology/C. W. Fetter.

p. cm.

Includes bibliographical references and index.

ISBN 0-02-337135-8

1. Water, Underground—Pollution. 2. Water, Underground—Pollution—United States.  
3. Transport theory. 4. Hydrogeology.

I. Title.

TD426.F48 1992

628.1'68—dc20

92-17787  
CIP

Printing: 2 3 4 5 6 7 8 9 Year: 3 4 5 6 7

Thus values of  $\operatorname{erfc}(B)$  range from 0 to +2, since the maximum value of  $\operatorname{erf}(B)$  is 1.0 for 3.0 and all greater numbers.

**EXAMPLE  
PROBLEM**

Assume a  $D$  of  $1 \times 10^{-9}$  m<sup>2</sup>/sec and an  $\omega$  of 0.5, to give a  $D^*$  of  $5 \times 10^{-10}$  m<sup>2</sup>/sec. Find the value of the concentration ratio,  $C_t/C_0$ , at a distance of 5 m after 100 yr of diffusion.

1. Convert 100 yr to seconds:

$$100 \text{ yr} \times 365 \text{ days/yr} \times 1440 \text{ min/day} \times 60 \text{ sec/min} = 3.15 \times 10^9 \text{ sec}$$

2. Insert values into Equation 2.5:

$$\frac{C_t}{C_0} = \operatorname{erfc} \frac{5}{2(5 \times 10^{-10} \text{ m}^2/\text{sec} \times 3.15 \times 10^9 \text{ sec})^{0.5}}$$

3. Solve:

$$\frac{C_t}{C_0} = \operatorname{erfc} \left( \frac{5}{2.51} \right) = \operatorname{erfc} 1.99 = 0.005$$

In 100 yr, diffusion over a 5-m distance would yield a concentration that is 0.5% of the original.

From the preceding example problem it is obvious that diffusion is not a particularly rapid means of transporting dissolved solutes. Diffusion is the predominant mechanism of transport only in low-permeability hydrogeologic regimes. However, it is possible for solutes to move through a porous or a fractured medium by diffusion even if the ground water is not flowing.

### 2.3 Transport by Advection

Dissolved solids are carried along with the flowing ground water. This process is called **advective transport**, or **convection**. The amount of solute that is being transported is a function of its concentration in the ground water and the quantity of the ground water flowing. For one-dimensional flow normal to a unit cross-sectional area of the porous media, the quantity of water flowing is equal to the *average linear velocity* times the *effective porosity*. **Average linear velocity**,  $v_x$ , is the rate at which the flux of water across the unit cross-sectional area of pore space occurs. It is not the average rate at which the water molecules are moving along individual flowpaths, which is greater than the average linear velocity due to tortuosity. The **effective porosity**,  $n_e$ , is the porosity through which flow can occur. Noninterconnected and dead-end pores are not included in the effective porosity.

$$v_x = \frac{K}{n_e} \frac{dh}{dl} \quad (2.6)$$

where

$v_x$  = average linear velocity (L/T)

$K$  = hydraulic conductivity (L/T)

$n_e$  = effective porosity

$db/dl$  = hydraulic gradient (L/L)

The one-dimensional mass flux,  $F_x$ , due to advection is equal to the quantity of water flowing times the concentration of dissolved solids and is given by Equation 2.7:

$$F_x = v_x n_e C \quad (2.7)$$

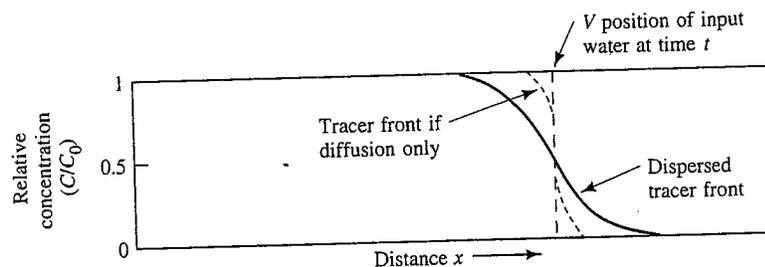
The one-dimensional advective transport equation is

$$\frac{\partial C}{\partial t} = -v_x \frac{\partial C}{\partial x} \quad (2.8)$$

(The derivation of this equation is given in Section 2.6.)

Solution of the advective transport equation yields a sharp concentration front. On the advancing side of the front, the concentration is equal to that of the invading ground water, whereas on the other side of the front it is unchanged from the background value. This is known as **plug flow**, with all the pore fluid being replaced by the invading solute front. The sharp interface that results from plug flow is shown in Figure 2.3. The vertical dashed line at  $V$  represents an advancing solute front due to advection alone.

Due to the heterogeneity of geologic materials, advective transport in different strata can result in solute fronts spreading at different rates in each strata. If one obtains a sample of water for purposes of monitoring the spread of a dissolved contaminant from a borehole that penetrates several strata, the water sample will be a composite of the water from each strata. Due to the fact that advection will transport solutes at different rates in each stratum, the composite sample may be a mixture of water containing the transported solute coming from one stratum and uncontaminated ground water coming from a different stratum where the average linear velocity is lower. The concentration of the contaminant in the composite sample would thus be less than in the source.



**FIGURE 2.3** Advective transport and the influence of longitudinal dispersion and diffusion on the transport of a solute in one-dimensional flow. Source: C. W. Fetter, *Applied Hydrogeology*, 2d ed. (New York: Macmillan Publishing Company, 1988).

equation  $\alpha_m = 0.0175L_s^{1.46}$ . Eventually the apparent dispersivity appears to reach a maximum value.

Stochastic methods of analysis have also been developed to analyze solute transport at the field scale. Stochastic methods are based on the variation in the hydraulic conductivity values because it is that variation that causes the solute plume to spread. The ground-water velocity depends upon the porosity as well as the hydraulic conductivity, but the hydraulic conductivity varies over a much greater range than porosity.

At the field scale the spreading due to hydraulic conductivity variation is much greater than that due to pore-scale dispersion. Both stochastic and advection-dispersion models demonstrate that the primary movement of the solute plume is due to advection. The stochastic model yields the movement of the center of mass of the solute plume from the average rate of movement of the ground water. The variance of the solute concentration about the mean position, or the second spatial moment, is also obtained from stochastic models.

If one has sufficient knowledge of the distribution of hydraulic conductivity in an aquifer, then a numerical advection-dispersion model of ground-water flow can be developed that uses a pore-scale dispersion value. This type of model has theoretical validity, because the necessary coefficient of longitudinal dispersion does not change with flow path length. It can be used to predict future solute concentrations at specific places and times. Naturally, such predictions will not be 100% accurate, because one can obviously never know the value of the hydraulic conductivity every place in the flow field.

### Chapter Notation

|                     |   |
|---------------------|---|
| $A$                 | Cross-sectional area  |
| $a$                 | Width of a fracture   |
| $b$                 | Aquifer thickness   |
| $B$                 | $[(v_x x)^2/(2D_L)^2 + (v_x y)^2/(4D_L D_T)]^{1/2}$                   |
| $C$                 | Solute concentration  |
| $C_i$               | Concentration at some point $x$ and time $t$                          |
| $C_0$               | Concentration at time 0   |
| $C_R$               | Dimensionless solute concentration ( $C/C_0$ )                        |
| $\langle C \rangle$ | Ensemble mean concentration   |
| $c_0$               | Constant related to anisotropy  |
| $d$                 | Characteristic flow length for Peclet number, $P$                     |
| $db/dl$             | Hydraulic gradient  |
| $D^*$               | Effective diffusion coefficient                                       |
| $D_d$               | Molecular diffusion coefficient                                       |
| $D_i$               | Coefficient of hydrodynamic dispersion in the $i$ direction           |
| $D_L$               | Coefficient of longitudinal hydrodynamic dispersion                   |
| $D_{LM}$            | Coefficient of longitudinal macrodispersivity at the asymptotic limit |
| $D_m$               | Field-measured (calculated) coefficient of hydrodynamic dispersion    |
| $D_T$               | Coefficient of transverse hydrodynamic dispersion                     |
| $E$                 | Euler number (0.577 . . .)  |
| $E_i$               | Exponential integral  |