Rock Slope Engineering

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Chapter 9 : Circular failure

Introduction

Although this book is concerned primarily with the stability of rock slopes, the reader will occasionally be faced with a slope problem involving soft materials such as overburden soils or crushed waste. In such materials, failure occurs along a surface which approaches a circular shape and this chapter is devoted to a brief discussion on how stability problems involving these materials are dealt with.

In a review on the historical development of slope stability theories, Golder[210] has traced the subject back almost 300 hundred years. During the past half century, a vast body of literature on this subject has accumulated and no attempt will be made to summarise this material in this chapter. Standard soil mechanics text books such as those by Taylor[174], Terzaghi[211] and Lambe and Whitman[212] all contain excellent chapters on the stability of soil slopes and it is suggested that at least one of these books should occupy a prominent place on the bookshelf of anyone who is concerned with slope stability. In addition to these books a number of important papers dealing with specific aspects of soil slope stability have been published and a selected list of these is given under references 213 to 233 at the end of this chapter.

The approach adopted in this chapter is to present a series of the slope stability charts for circular failure. These charts enable the user to carry out a very rapid check on the factor of safety of a slope or upon the sensitivity of the factor of safety to changes in groundwater conditions or slope profile. These charts should only be used for the analysis of circular failure in materials where the properties do not vary through the soil or waste rock mass and where the conditions assumed in deriving the charts, discussed in the next section, apply. A more elaborate form of analysis is presented at the end of this chapter for use in cases where the material properties vary within the slope or where part of the slide surface is at a soil/rock interface and the shape of the failure surface differs significantly from a simple circular arc.

Conditions for circular failure

In the previous chapters it has been assumed that the failure of rock slopes is controlled by geological features such as bedding planes and joints which divide the rock body up into a discontinuous mass. Under these conditions, the failure path is normally defined by one or more of the discontinuities. In the case of a soil, a strongly defined structural pattern no longer exists and the failure surface is free to find the line of least resistance through the slope. Observations of slope failures in soils suggests that this failure surface generally takes the form of a circle and most stability theories are based upon this observation.

The conditions under which circular failure will occur arise when the individual particles in a soil or rock mass are very small as compared with the size of the slope and when these particles are not interlocked as result of their shape. Hence, crushed rock in a large waste dump will tend to behave as a "soil" and large failures will occur in a circular mode. Alternatively, the finely ground waste

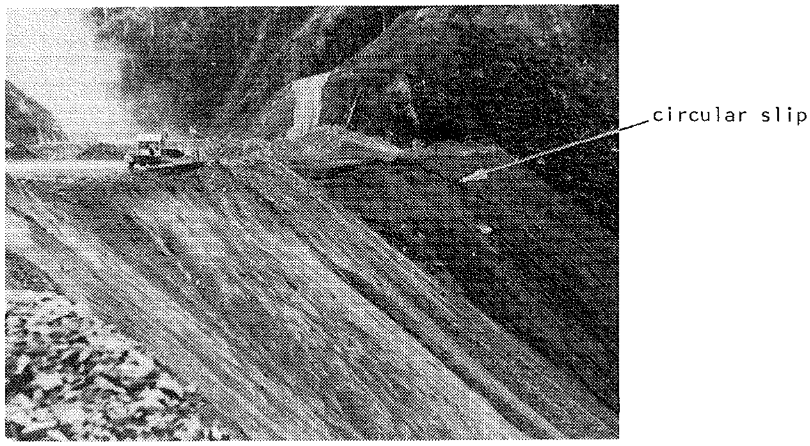

Figure 102 : Shallow surface failure in large waste dumps are generally of a circular type.

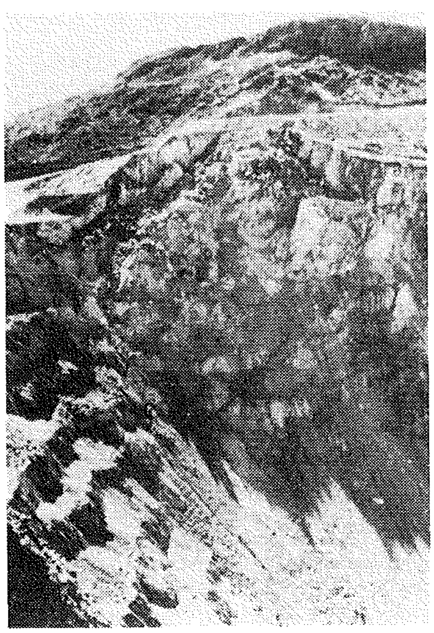

Figure 103 : Circular failure in the highly altered and weathered rock forming the upper benches of an open pit mine.

material which has to be disposed of after completion of a milling and metal recovery process will exhibit circular failure surfaces, even in slopes of only a few feet in height. Highly altered and weathered rocks will also tend to fail in this manner and it is appropriate to design the overburden slopes around an open pit mine on the assumption that failure would be by a circular failure process.

Derivation of circular failure charts

The following assumptions are made in deriving the stability charts presented in this chapter :

a. The material forming the slope is assumed to be homogeneous, i.e. its mechanical properties do not vary with direction of loading

b. The shear strength of the material is characterised by a cohesion c and a friction angle ϕ which are related by the equation $\tau = c + \sigma.\text{Tan}\phi$.

c. Failure is assumed to occur on a circular failure surface which passes through the toe of the slope*.

d. A vertical tension crack is assumed to occur in the upper surface or in the face of the slope.

e. The locations of the tension crack and of the failure surface are such that the factor of safety of the slope is a minimum for the slope geometry and groundwater conditions considered.

f. A range of groundwater conditions, varying from a dry slope to a fully saturated slope under heavy recharge, are considered in the analysis. These conditions are defined later in this chapter.

Defining the factor of safety of the slope as

$$F = \frac{\text{Shear strength available to resist sliding}}{\text{Shear stress mobilised along failure surface}}$$

and rearranging this equation, we get

$$\tau_{mb} = \frac{c}{F} + \frac{\sigma.\text{Tan}\phi}{F} \qquad (97)$$

where τ_{mb} is the shear stress mobilised along the failure surface.

Since the shear strength available to resist sliding is dependent upon the distribution of the normal stress σ along this surface and, since this normal stress distribution is unknown, the problem is statically indeterminate. In order to obtain a solution it is necessary to assume a specific normal stress distribution and then to check whether this distribution gives meaningful practical results.

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*Terzaghi[211], page 170, shows that the toe failure assumed for this analysis gives the lowest factor of safety provided that $\phi > 5^{o}$. The $\phi = 0$ analysis, involving failure below the toe of the slope through the base material has been discussed by Skempton[234] and by Bishop and Bjerrum[235] and is applicable to failures which occur during or after the rapid construction of a slope. Such conditions are unlikely to occur in typical mining operations.

The influence of various normal stress distributions upon the factor of safety of soil slopes has been examined by Frohlich[216] who found that a *lower bound* for all factors of safety which satisfy statics is given by the assumption that the normal stress is concentrated at a single point on the failure surface. Similarly, the *upper bound* is obtained by assuming that the normal load is concentrated at the two end points of the failure arc.

The unreal nature of these stress distributions is of no consequence since the object of the exercise, up to this point, is simply to determine the extremes between which the actual factor of safety of the slope must lie. In an example considered by Lambe and Whitman[212], the upper and lower bounds for the factor of safety of a particular slope corresponded to 1.62 and 1.27 respectively. Analysis of the same problem by Bishop's simplified method of slices gives a factor of safety of 1.30 which suggests that the actual factor of safety may lie reasonably close to the lower bound solution.

Further evidence that the lower bound solution is also a meaningful practical solution is provided by an examination of the analysis which assumes that the failure surface has the form of a logarithmic spiral[227]. In this case, the factor of safety is independent of the normal stress distribution and the upper and lower bounds coincide. Taylor[174] compared the results from a number of logarithmic spiral analyses with results of lower bound solutions* and found that the difference is negligible. On the basis of this comparison, Taylor concluded that the lower bound solution provides a value of the factor of safety which is sufficiently accurate for most practical problems involving simple circular failure of homogeneous slopes.

The authors have carried out similar checks to those carried out by Taylor and have reached the same conclusions. Hence, the charts presented in this chapter correspond to the lower bound solution for the factor of safety, obtained by assuming that the normal load is concentrated at a single point on the failure surface. These charts differ from those published by Taylor in 1948 in that they include the influence of a critical tension crack and of groundwater in the slope.

Groundwater flow assumptions

In order to calculate the uplift force due to water pressure acting on the failure surface and the force due to water in the tension crack, it is necessary to assume a set of groundwater flow patterns which coincide as closely as possible with those conditions which are believed to exist in the field.

In the analysis of rock slope failures, discussed in chapters 7 and 8, it was assumed that most of the water flow took place in discontinuities in the rock and that the rock itself was practically impermeable. In the case of slopes in soil or waste rock, the permeability of the mass of

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* The lower bound solution discussed in this chapter is usually known as the *Friction Circle Method* and was used by Taylor[174] for the derivation of his stability charts.

material is generally several orders of magnitude higher than that of intact rock and, hence, a general flow pattern will develop in the material behind the slope.

Figure 55a on page 137 shows that, within the soil mass, the equipotentials are approximately perpendicular to the phreatic surface. Consequently, the flow lines will be approximately parallel to the phreatic surface for the condition of steady state drawdown. Figure 104a shows that this approximation has been used for the analysis of the water pressure distribution in a slope under conditions of normal drawdown. Note that the phreatic surface is assumed to coincide with ground surface at a distance x, measured in multiples of the slope height, behind the toe of the slope. This may correspond to the position of a surface water source such as a river or dam or it may simply be the point where the phreatic surface is judged to intersect the ground surface.

The phreatic surface itself has been obtained, for the range of slope angles and values of x considered, by a computer solution of the equations proposed by L. Casagrande[236], discussed in the text book by Taylor[174].

For the case of a saturated slope subjected to heavy surface recharge, the equipotentials and the associated flow lines used in the stability analysis are based upon the work of Han[237] who used an electrical resistance analogue method for the study of groundwater flow patterns in isotropic slopes.

Production of circular failure charts

The circular failure charts presented in this chapter were produced by means of a Hewlett-Packard 9100 B calculator with graph plotting facilities. This machine was programmed to seek out the most critical combination of failure surface and tension crack for each of a range of slope geometries and groundwater conditions. Provision was made for the tension crack to be located in either the upper surface of the slope or in the face of the slope. Detailed checks were carried out in the region surrounding the toe of the slope where the curvature of the equipotentials results in local flow which differs from that illustrated in Figure 104a.

The charts are numbered 1 to 5 to correspond with the groundwater conditions defined in the table presented on page 233.

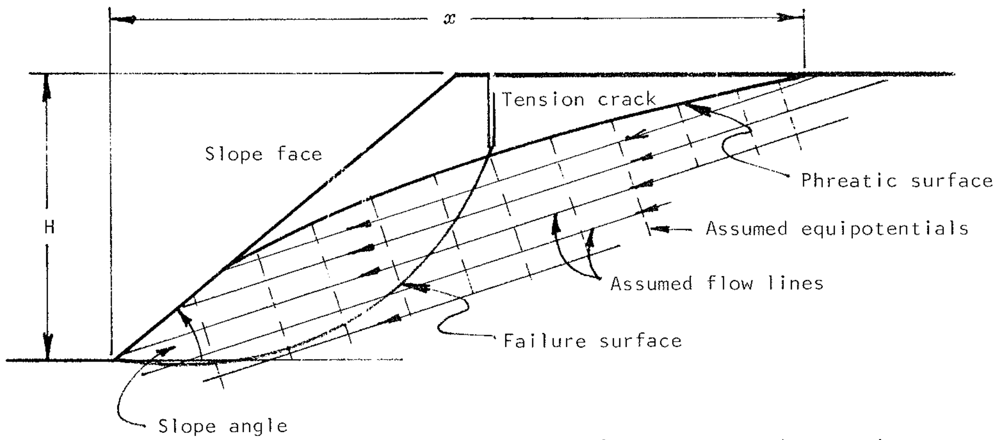
Use of the circular failure charts

In order to use the charts to determine the factor of safety of a particular slope, the steps outlined below and shown in Figure 105 should be followed.

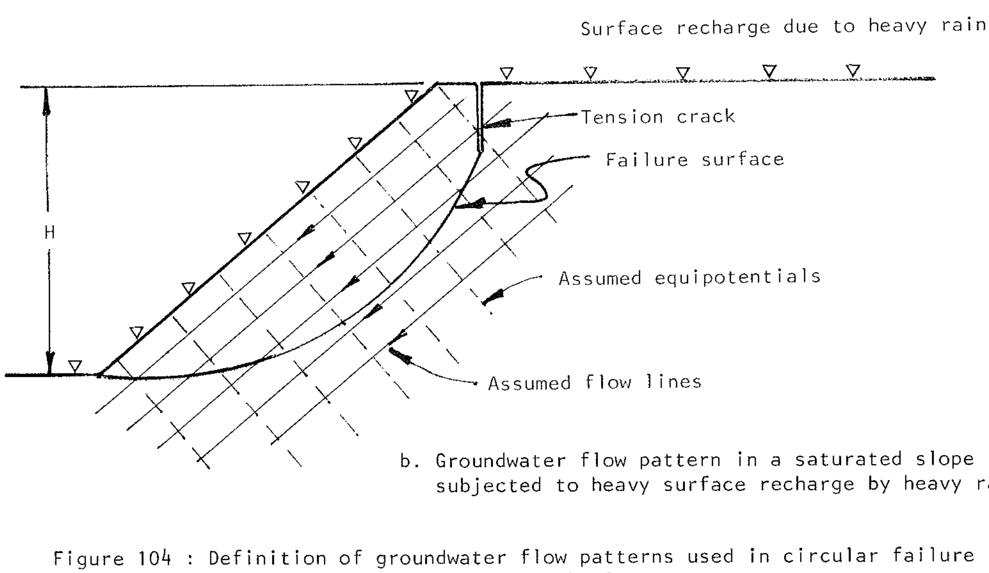
Step 1 : Decide upon the groundwater conditions which are believed to exist in the slope and choose the chart which is closest to these conditions, using the table presented on page 233.

Step 2 : Calculate the value of the dimensionless ratio

$$\frac{c}{\gamma H.\tan\phi}$$



a. Groundwater flow pattern under steady state drawdown conditions where the phreatic surface coincides with the ground surface at a distance x behind the toe of the slope. The distance x is measured in multiples of the slope height H.



b. Groundwater flow pattern in a saturated slope subjected to heavy surface recharge by heavy rain.

Figure 104 : Definition of groundwater flow patterns used in circular failure analysis of soil and waste rock slopes.

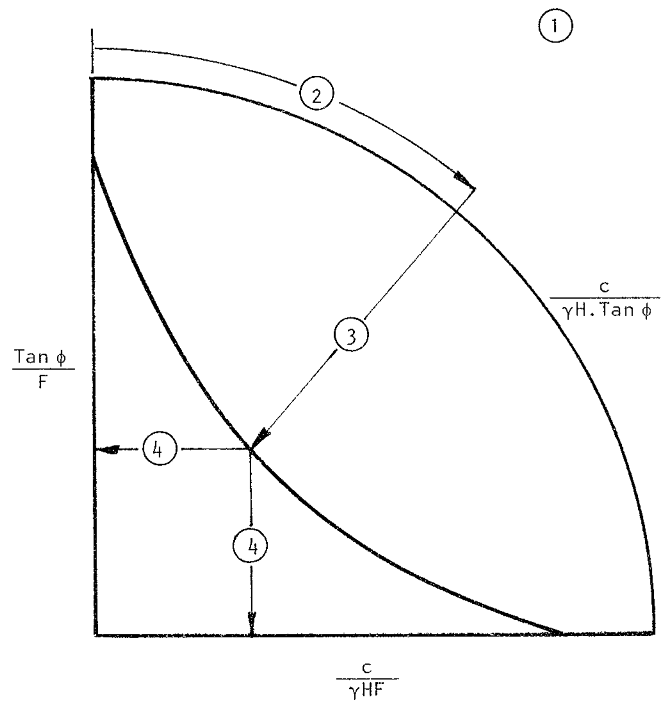


Figure 105 : Sequence of steps involved in using circular failure charts to find the factor of safety of a slope.

Find this value on the outer circular scale of the chart.

Step 3 : Follow the radial line from the value found in step 2 to its intersection with the curve which corresponds to the slope angle under consideration.

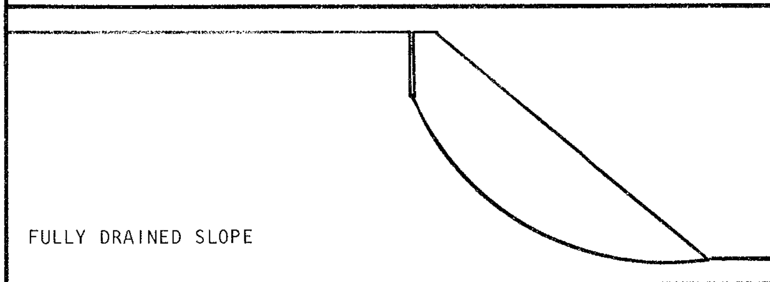
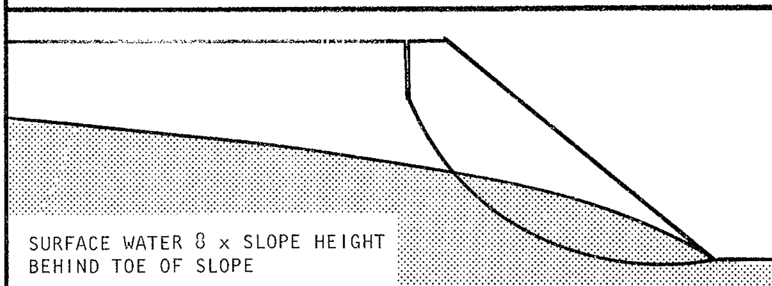
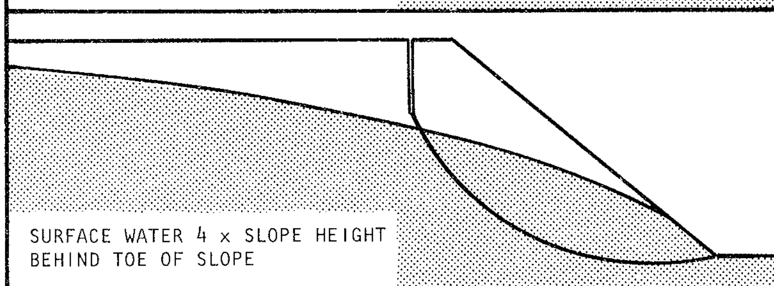
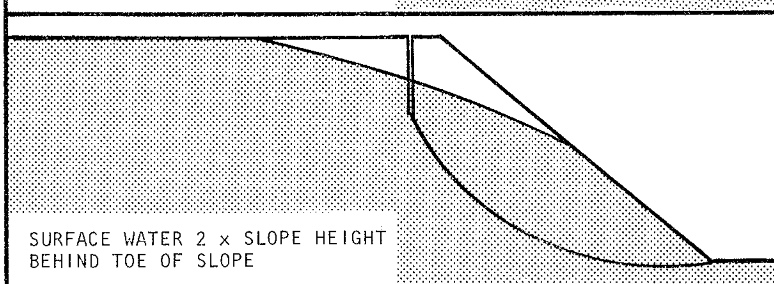
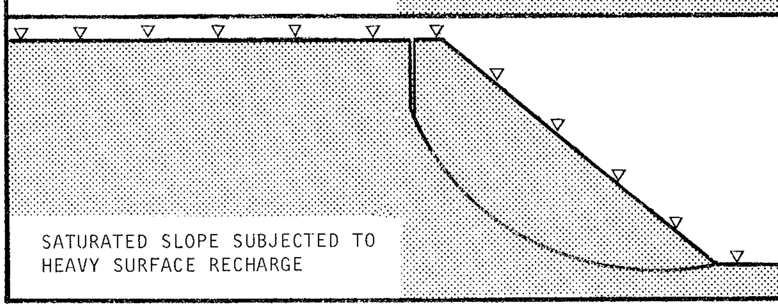
Step 4 : Find the corresponding value of Tanϕ/F or c/γHF, depending upon which is more convenient, and calculate the factor of safety.

Consider the following example :

A 50 foot high slope with a face angle of 40^o is to be excavated in overburden soil with a density γ = 100 lb/ft^3, a cohesive strength of 800 lb/ft^2 and a friction angle of 30^o. Find the factor of safety of the slope, assuming that there is a surface water source 200 feet behind the toe of the slope.

The groundwater conditions indicate the use of chart No.3. The value of c/γH.Tanϕ = 0.28 and the corresponding value of Tanϕ/F, for a 40^o slope, is 0.32. Hence, the factor of safety of the slope is 1.80.

Because of the speed and simplicity of using these charts, they are ideal for checking the sensitivity of the factor of safety of a slope to a wide range of conditions and the authors suggest that this should be their main use.

GROUNDWATER FLOW CONDITIONS	CHART NUMBER
 FULLY DRAINED SLOPE	1
 SURFACE WATER 8 x SLOPE HEIGHT BEHIND TOE OF SLOPE	2
 SURFACE WATER 4 x SLOPE HEIGHT BEHIND TOE OF SLOPE	3
 SURFACE WATER 2 x SLOPE HEIGHT BEHIND TOE OF SLOPE	4
 SATURATED SLOPE SUBJECTED TO HEAVY SURFACE RECHARGE	5

CIRCULAR FAILURE CHART NUMBER 1